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ON INTERACTION OF ACOUSTIC AND VORTEX PERTURBATIONS IN A BOUNDED PLANE PARALLEL VISCOUS GAS FLOW

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Linearized Navier-Stokes equations are used to study the propagation of acoustic and vortex perturbation in a gas flow along a channel of finite length. A system of equations for the perturbed pressure and vorticity is obtained under the assumption that the oscillations are isentropic.

The properties of the gas are assumed constant throughout the volume, and the lack of uniformity in the flow is connected with the velocity gradient only.

Let us consider the isentropic oscillations. Variation in the entropy will also be generated in the gradient flow of the viscous gas, however the entropic perturbations need not be taken into account, since, unlike the acoustic perturbations, they can only propagate downstream. The small isentropic variations in the gas density ρ_1 and pressure p_1 are connected by the following relation:

$$\eta \equiv \frac{\rho_1}{\rho} = \frac{p_1}{(\rho a^2)}$$

The linearized equations of small perturbations in the velocity $\mathbf{v}\,(v_x, v_y, v_z)$ and pressure η have the form

$$\frac{\partial \mathbf{v}}{\partial t} + U\frac{\partial \mathbf{v}}{\partial x} + v_y \frac{dU}{dy}\mathbf{e}_x = -a^2\nabla\eta + \nu\Delta\mathbf{v} + \frac{\nu}{3}\nabla \operatorname{div}\mathbf{v} \tag{1}$$
$$\frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial x} + \operatorname{div}\mathbf{v} = 0$$

Here $U(y)$ denotes the steady velocity of the unperturbed flow along the x-axis; ρ, ν and a denote the constant density and viscosity of the gas and the speed of sound, respectively; and $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ are the components of the unit vector.

The interdependence of the vortical and acoustic oscillations can be shown simply by deriving from (1) the equations for the perturbed vorticity $\boldsymbol{\omega} = \operatorname{rot}\mathbf{v}$ and pressure η. Taking rot and div of the first equation of (1), we obtain for $\boldsymbol{\omega}$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + U\frac{\partial \boldsymbol{\omega}}{\partial x} - \nu\Delta\boldsymbol{\omega} = \left[v_y \frac{d^2U}{dy^2} + \frac{dU}{dy}\left(\operatorname{div}\mathbf{v} - \frac{\partial v_z}{\partial z}\right)\right]\mathbf{e}_z - \frac{dU}{dy}\left(\frac{\partial v_z}{\partial x}\mathbf{e}_x + \frac{\partial v_y}{\partial z}\mathbf{e}_y\right) \tag{2}$$

For the components ω_x and ω_y the sources of vorticity proportional to dU/dy exist only in the case of three-dimensional perturbations.

The variation in ω_z is described by the equation

$$\frac{\partial \omega_z}{\partial t} + U\frac{\partial \omega_z}{\partial x} - \nu\Delta\omega_z = v_y \frac{d^2U}{dy^2} + \frac{dU}{dy}\left(\operatorname{div}\mathbf{v} - \frac{\partial v_z}{\partial z}\right) \tag{3}$$

The dependence of the vorticity on the acoustic oscillations is given not only by the term proportional to div v; the velocity v_y is determined by both $\boldsymbol{\omega}$ and div v.

We have the following equation for the dimensionless pressure η :

$$\frac{\partial^2\eta}{\partial t} + 2U\frac{\partial^2\eta}{\partial t\partial x} + U^2\frac{\partial^2\eta}{\partial x^2} - \left[a^2 + \frac{4}{3}\nu\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\right]\triangle\eta = \tag{4}$$
$$2\frac{\partial v_y}{\partial x}\frac{dU}{dy} + \frac{4}{3}\nu\left(2\frac{dU}{dy}\frac{\partial^2\eta}{\partial x\partial y} + \frac{d^2U}{dy^2}\frac{\partial\eta}{\partial x}\right)$$

The sources of the acoustic oscillations in the gradient flow appear in the right-hand part.

To eliminate the velocity v_y appearing in (3) and (4), we must use the expressions for $\boldsymbol{\omega}$ and div v as well as the second equation of (1).

When div v $= 0$, the velocity v_y is expressed in terms of $\boldsymbol{\omega}$ and Eq. (3) for the plane oscillations reduces to the known Orr – Sommerfeld equation containing the first time derivative. In the general case Eqs.(3) and (4) contain the first, second and third time derivatives and the solution contains three interrelated waves: the forward and reverse acoustic waves, and the Tollmin – Schlichting wave. The phase velocities of the first two waves are nearly sonic (with a correction for drift due to the stream) and velocity of the third wave lies within the range of variation of the steady state stream velocity. The above waves will naturally be interconnected by the boundary conditions. The acoustic waves propagating upstream in a subsonic flow may cause absolute instability in a bounded flow. We give one exact relation connecting the acoustic plane perturbations with the vortical perturbations, which follows from the expression (3) for a flow with a linear velocity profile and viscous term neglected

$$\omega_z / \Omega_0 + \eta = \text{const}$$

Here $\Omega_0 = -dU / dy$ is the constant vorticity of the steady flow. This invariant is obtained by substituting the expression for div v from the second equation of (1) into (3) in which $\nu = 0$ and $d^2U / dy^2 = 0$.

Translated by L. K.